

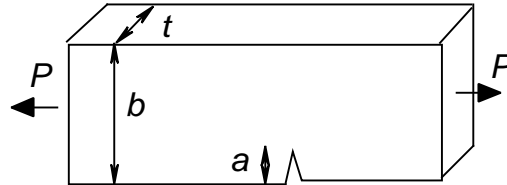
ANOTHER EXAMPLE OF FRACTURE-MECHANICS

Always check for yield and fracture

- Rectangular plate ($b = 50 \text{ mm} \times t = 10 \text{ mm}$) with single edge crack (length a)

Made of aluminum alloy ($K_{Ic} = 34 \text{ MPa} \cdot \text{m}$, $\sigma_Y = 325 \text{ MPa}$)

Calculate failure load for different values of a



For this geometry: Remote stress: $\sigma = P/bt$
 In plane of crack: $\sigma_1 = \frac{P}{(b-a)t}$; $\sigma_2 = 0$; $\sigma_3 = 0$

(1) Consider yield

- Yield will occur first in plane of crack

Loading parameter for yield: $\tilde{\sigma}_H = \frac{P}{(b-a)t}$

- Material property for yield: σ_Y (325 MPa)
- Yield criterion: $\tilde{\sigma}_H \leq \sigma_Y$

$$\begin{aligned} \text{Load for yield: } P_Y &= \sigma_Y (b-a)t = 325 \times 10^6 (0.050 - a) \times 10^{-2} \\ &= 3.25 (0.050 - a) \text{ MN} \quad (\text{if } a \text{ in m}) \end{aligned}$$

(2) Consider fracture:

- Loading parameter for fracture: $K_I = f(a/b)\sigma \sqrt{\pi a} = f(a/b) \frac{P}{bt} \sqrt{\pi a}$
 where $f(a/b)$ given in Dowling or data book

- Material property for fracture: K_{Ic} (34 MPa \cdot m)
- Fracture criterion: $K_I \leq K_{Ic}$

$$\text{Load for fracture: } P_F = \frac{b t K_{Ic}}{f(a/b) \sqrt{\pi a}} = \frac{(5 \times 10^{-2})(10^{-2}) \times 34 \times 10^6}{f(a/b) \sqrt{\pi a}}$$

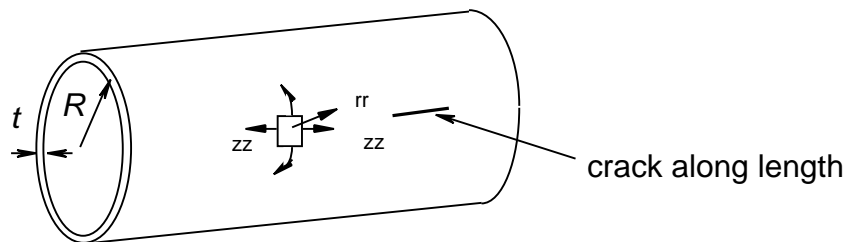
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$$= \frac{9.591}{f(a/b)\sqrt{a}} \text{ kN} \quad (\text{if } a \text{ in m})$$

a (m)	$(b-a)$ m	a/b	$f(a/b)$	P_Y (kN)	P_F (kN)	P_{Failure} (kN)
0	0.050	0	1.12	163		160
0.0025	0.0475	0.05	1.16	154	165	150
0.005	0.045	0.1	1.25	146	109	110
0.010	0.040	0.2	1.37	130	70.0	70
0.020	0.030	0.4	2.11	97.5	32.1	32

DESIGN OF PRESSURE VESSELS

- Cylindrical pressure vessel: $r \gg t$;
- Material parameters: Yield stress: σ_Y ; Fracture toughness: K_{Ic}



- Principal stresses: $\sigma_{rr} = -P/2$, $\sigma_{\theta\theta} = PR/t$; $\sigma_{zz} = PR/2t$

(1) Consider yield

- Loading parameter for yield: Von Mises effective stress:

$$\tilde{\sigma}_H = \frac{1}{2} \left(\frac{PR}{t} \right)^2 + \frac{PR}{2t} \left(\frac{PR}{2t} \right)^2 \quad \frac{\sqrt{3}}{2} \frac{PR}{t}$$

- Material property for yield: σ_Y
- Yield criterion: $\tilde{\sigma}_H \leq \sigma_Y$

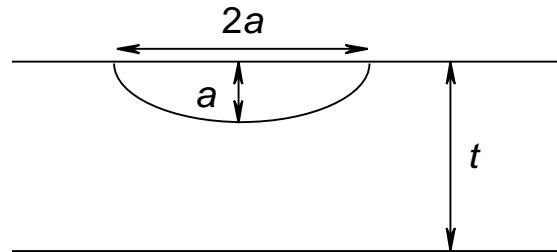
$$\text{Design against yield:} \quad \frac{PR}{t} < \frac{2}{\sqrt{3}} \sigma_Y \quad (\text{note: no } \sigma_{zz} \text{!})$$

(2) Consider fracture:

- Hoop stress, $\sigma_{\theta\theta}$, is largest, therefore axial cracks are most serious

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- Pipes and cylinders generally explode by splitting along their length



- Loading parameter for fracture: K_I

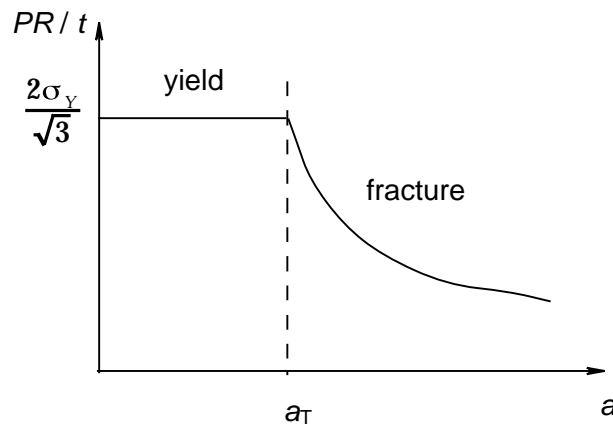
- Can assume that $K_I = \sigma_{\infty} \sqrt{\pi a}$

$$K_I = \frac{PR}{t} \sqrt{\pi a}$$

- Material property for fracture: K_{Ic}

- Fracture criterion: $K_I = K_{Ic}$

Design against fracture: $\frac{PR}{t} < \frac{K_{Ic}}{\sqrt{\pi a}}$



- Transition crack length, a_T , when $\left. \frac{PR}{t} \right|_{\text{yield}} = \left. \frac{PR}{t} \right|_{\text{fracture}}$

$$\frac{2}{\sqrt{3}} \sigma_Y = \frac{K_{Ic}}{\sqrt{\pi a}}$$

$$a_T = \frac{3}{4\pi} \left(\frac{K_{Ic}}{\sigma_Y} \right)^2$$

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Example 1: Pressure-vessel steel ASTM A517-F ($K_{Ic} = 187 \text{ MPa} \cdot \text{m}$, $\sigma_Y = 760 \text{ MPa}$)

$$a_T = 14 \text{ mm}$$

Example 2: 2014 Aluminum alloy ($K_{Ic} = 24 \text{ MPa} \cdot \text{m}$, $\sigma_Y = 415 \text{ MPa}$)

$$a_T = 0.8 \text{ mm}$$

- Much easier to detect a 14 mm long crack than a 1 mm long crack
- Yield is a “safer” failure mechanism than fracture

one design philosophy: make transition crack length as large as possible

- A safer design criterion is “Leak before break”

“Leak-before-break” design for pressure vessels

- Assume cracks are approximately semi-circular

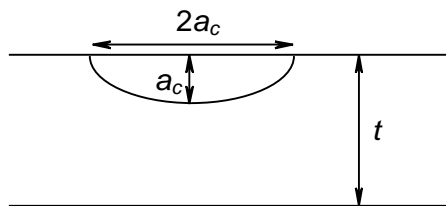
$$K_I = \sigma_{\theta\theta} \sqrt{\pi a}, \quad \text{where } \sigma_{\theta\theta} = PR/t$$

Fracture occurs when crack length is given by

$$a = a_c = \frac{1}{\pi} \left(\frac{K_{Ic} t}{Pr} \right)^2$$

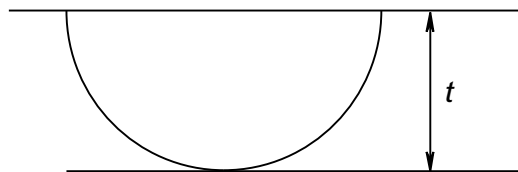
- Two possibilities:

(i) $a_c < t$



Explosion occurs

(ii) $a_c > t$



Pressure vessel will leak before explosion

- Leak-before-break criterion: $a_c > t$

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$$\frac{1}{\pi} \frac{K_{Ic} t^2}{PR} > t$$

$$\frac{PR}{\sqrt{t}} < \frac{K_{Ic}}{\sqrt{\pi}}$$

- Must also remember to design against yield: $\frac{PR}{t} < \frac{2\sigma_Y}{\sqrt{3}}$

Don't waste time remembering these equations - derive them if needed

Example: Thickness of pressure vessel with $P = 1.0$ MPa, $R = 1.0$ m, and safety factor of 3.0 on the thickness.

Use a 2014 Aluminum alloy ($K_{Ic} = 24$ MPa m, $\sigma_Y = 415$ MPa).

1. Design against yield: $\frac{PR}{t} < \frac{2\sigma_Y}{\sqrt{3}}$

$$t > \frac{\sqrt{3}PR}{2\sigma_Y}$$

$$t > \frac{10^6 \times 1 \times \sqrt{3}}{2 \times 415 \times 10^6} = 3.1 \text{ mm}$$

With safety factor

$$t = 9.3 \text{ mm}$$

2. Design for “leak-before-break”: $\frac{PR}{\sqrt{t}} < \frac{K_{Ic}}{\sqrt{\pi}}$

$$t > \pi \frac{PR^2}{K_{Ic}^2}$$

$$t > \pi \frac{10^6 \times 1}{24 \times 10^6}^2$$

$$t > 1.7 \text{ mm}$$

With safety factor

$$t = 5.1 \text{ mm}$$

Minimum thickness is 9.3 mm